Analytical modeling of deformable porous composites

F.-X. Bécot, L. Jaouen, F. Chevillotte
MATELYS - Acoustique & Vibrations, 1 rue Baumer, F-69120 Vaulx-en-Velin, France.

Summary
This paper is concerned with the analytical modeling of the acoustic properties of materials which could show interesting combined performances of sound absorption and sound insulation. These materials, made from porous materials having porous inclusions, are called composite porous materials by analogy with standard composite materials composed by a series of solid (non-porous) fibers embedded in a solid (usually non-porous) matrix. The efforts are concentrated here on the analytical modeling of the visco-thermal and structural dissipations which occur in the fluid phase and the solid phase respectively. Using dedicated experimental techniques for the characterisation of the porous components, the approach is validated against measured data obtained in impedance tubes. This work finds important applications in the field of the vibro-acoustic properties of sound packages which should combine performances both in terms of sound absorption and sound insulation.

PACS no. 43.20.Gp, 43.20.Jr, 43.55.Ev

1. Introduction

It was demonstrated recently that multi-scale porous materials could be used as efficient sound absorbing treatments. Double porosity or perforated porous materials [1] are examples of such materials as well as porous composites materials obtained by filling the perforations with a porous material of different nature [2] (see Figure 1 for examples of porous composites). Comparatively, whereas sound insulation properties of double porosity materials are low [3], materials of the second category were proved to offer interesting combined properties of both sound absorption and sound insulation [4].

Current predictive models for such media are limited by the assumption that the porous frames are rigid and motionless. This restrains the range of applications which could be addressed [2]. In fact, when porous materials are coupled to elastic structures, deformations of the porous frame need to be taken into account to predict accurately the response of the system. The present work aims at overcoming this lack.

The proposed analytical model is based on Biot’s theory [5] and the underlying assumption that the porous frame deformation does not influence the dissipation occurring inside the fluid. Accordingly, the proposed model handles separately the structural dissipation in the porous frame and the visco-inertial dissipation in the fluid.

The acoustic wave propagation is modeled using the derivation of Olny & Boutin theory [6, 7, 8] for rigid framed double porosity materials. The elastic response of the composite porous is modeled using a self consistent scheme for solid composites [9, 10, 11]. These two dissipation mechanisms are coupled using

(c) European Acoustics Association
a formulation of Biot’s poroelasticity equations using a \((u^s, p)\) formalism \([12, 13]\). The complete model is finally implemented in Transfer Matrix (TMM) algorithm \([14]\) to associate various layers of different types: impervious films, fabrics, porous or elastic (non-porous) materials.

The paper is organised as follows. The theory of both the fluid and structural dissipation is briefly described in the following two paragraphs. Simulation results are validated against impedance tube measurements of sound absorption and sound transmission coefficients for various configurations.

2. Analytical modeling of the poroelastic medium

Porous composite materials are above all poroelastic (porous) materials. They are composed of a solid phase, the porous skeleton, and a fluid phase, the air contained inside the pores.

The coupling of the fluid and the solid phase has been proposed by Biot in [5]. This formalism provides the possibility to model the dissipation inside the fluid separately from the modeling of the structural dissipation in the porous frame. Initially expressed in terms of the two displacements of the solid and the fluid phase, works in [12, 13] proposed a formalism in terms of the solid phase displacements \(u^s\) and the interstitial pressure \(p\). The two coupled poroelasticity equations then write:

\[
\nabla \tilde{\sigma} + \omega^2 \tilde{\rho} u^s = \tilde{\gamma} \nabla p \\
\n\nabla . \tilde{\sigma} + \omega^2 \tilde{\rho} u^s + \frac{\tilde{\rho}_{22}}{\tilde{R}} \omega^2 p = \frac{\tilde{\rho}_{22}}{\tilde{\phi}^2} \tilde{\gamma} \omega^2 \nabla \cdot u^s
\]

In this equation, the superscript “\(\tilde{\cdot}\)" indicates that the variable is frequency dependent.

\(\tilde{\sigma}\) is the in-vacuo stress tensor of the solid phase. In addition, \(\tilde{\rho}, \tilde{\rho}_{11}, \tilde{\rho}_{22}\) and \(\tilde{\rho}_{12}\) are the modified Biot’s densities the expressions of which are given below. \(\tilde{\gamma}\) is a coupling factor given by \(\tilde{\gamma} = \phi (\tilde{\rho}_{12}/\tilde{\rho}_{22} - \tilde{Q}/\tilde{R})\). \(\tilde{Q}\) is a factor which couples the skeleton strain to the fluid strain, and \(\tilde{R}\) can be interpreted as the bulk modulus of a volume of fluid occupying a fraction \(\phi_{\text{mat}}\) of the porous media, \(\phi_{\text{mat}}\) being the porosity of the medium.

The above parameters relate to the porous material characteristics in the following manner:

\[
\tilde{\rho} = \tilde{\rho}_{11} - \tilde{\rho}_{12}/\tilde{\rho}_{22} \\
\tilde{\rho}_{11} = (1 - \phi_{\text{mat}}) \rho_1 - \phi_{\text{mat}} \rho_0 (\phi_{\text{mat}} - 1) \\
\tilde{\rho}_{12} = \rho_1 - \tilde{\rho}_{22} \\
\tilde{\rho}_{22} = \phi_{\text{mat}}^2 \rho_{\text{mat}} \\
\tilde{Q} = (1 - \phi_{\text{mat}}) \tilde{K}_{\text{mat}} \\
\tilde{R} = \phi_{\text{mat}} \tilde{K}_{\text{mat}}
\]

In these equations, \(\rho_1\) is the mass density of the porous material and \(\rho_0\) is the mass density of the air saturating the porous medium. In addition, \(\tilde{\alpha}_{\text{mat}}, \tilde{\rho}_{\text{mat}}\) and \(\tilde{K}_{\text{mat}}\) are the dynamic tortuosity, the dynamic mass density and dynamic bulk modulus of the porous material (see also Section 3).

Note that the last two relationships only hold when the bulk modulus of the solid phase is very large compared to that of the fluid phase and to that of the solid phase in-vacuo. This condition generally applies for the poroelastic materials encountered in most of sound packages. For interested readers, expressions valid for any type of porous materials are available and can be found in [15].

Once these parameters are determined, the equations of motions are implemented in a TMM algorithm [14] and the response in both sound absorption or sound transmission for any kind of multi-layer systems can be computed.

Therefore, the problematic is to express the effective properties of both the solid phase and of the fluid phase for the porous composite material. For the fluid phase, since the propagation of sound wave inside the pores is dispersive, the effective properties of interest here are the dynamic mass density \(\tilde{\rho}_{\text{mat}}\) and the dynamic bulk modulus \(\tilde{K}_{\text{mat}}\). For the solid phase, if we assume that the material is isotropic, the effective properties are Young’s modulus \(E\), Poisson’s ratio \(\nu\), structural loss factor \(\eta\) and the mass density \(\rho_1\). This assumption will be discussed later in the paper.

The following two sections present the separate computation of these effective properties.

3. Modeling of the fluid dissipation

The calculation of the dissipation inside the fluid corresponds to the computation of the effective properties of a rigid and motionless porous material. It is based on the theory of double porosity materials which allows the analytical modeling of a perforated porous material [8].

In the case of a porous composite, the perforation is filled with another porous material (see Figure 1), which will be called the Client material. The porous material substrate hosting the Client will be called the Host material. An analytical model has been proposed in [2] for this type of material. The idea is to model separately the visco-thermal in the Client and in the Host as if they were infinite media. Then, the two viscous permeabilities, respectively the two bulk moduli, are coupled to represent the visco-inertial effects, respectively the thermal effects, as proposed in the double porosity theory. By doing so, the coupling is a “volumic” coupling in the sense that the main governing parameter is the rate of inclusion, also called meso-porosity and denoted \(\phi_p\). In other words, the information about the shape of the inclusion is lost.

To overcome this, the approach used here is slightly different. The coupling of the visco-inertial effects is still realised via the coupling of the permeabilities, as done in [2]. The first difference relies on the fact that
the shape of the inclusion is accounted for. In this work, only cylindrical inclusions are considered. The second difference is that the coupling of the thermal effects is realised via the coupling of the thermal permeabilities instead of the coupling of the bulk moduli.

In the absence of analytical solution for the permeabilities of a porous cylinder, the viscous and thermal permeabilities, indexed C for Client, of the cylindrical porous inclusion are modeled on the basis of the sound propagation in a cylinder filled with fluid. We propose here to use the following expressions:

\[ \Pi_C = j\delta_t^2 \left[ 1 - \frac{2}{\mu_\nu \sqrt{\xi}} J_1(\mu_\nu \sqrt{\xi}) \right] \]
\[ \Theta_C = j\delta_t^2 \left[ 1 - \frac{2}{\mu_\nu \sqrt{\xi}} J_1(\mu_\nu \sqrt{\xi}) \right] \]

In these equations, \( \delta_t = \sqrt{\eta_0/\rho_0 \omega} \) and \( \delta_t = \sqrt{\eta_0/Pr \omega} \) are the dynamic viscous and thermal skin depths, where \( \eta_0 \) and \( \rho_0 \) are the viscosity and mass density of the air saturating the porous material. \( \mu_\nu = \Lambda/\delta_t \) and \( \mu_\nu = \Lambda'/\delta_t \) and \( J_1 \) and \( J_0 \) are Bessel function of the first kind of zero-th and first order. \( \Lambda \) and \( \Lambda' \) are the viscous and thermal characteristic lengths.

The viscous and thermal permeabilities for the Host material are computed assuming the medium is infinite. In a similar manner as done for double porosity materials, the coupling between the Host and the Client materials is ensured via the following relationships:

\[ \Pi_{pc} = \phi_p \Pi_C + (1 - \phi_p) \Pi_H \]
\[ \Theta_{pc} = \phi_p \Theta_C + (1 - \phi_p) \Theta_H \times F_d \]

where the subscript “pc” stands for Porous Composites. The viscous and thermal permeabilities \( \Pi_H \) and \( \Theta_H \) for the Host material are given by the model of Johnson-Champoux-Allard-Lafarge (JCAL) model [16, 17, 18].

Note that possible pressure diffusion effects may be accounted for at this stage of the modeling via the complex function \( F_d \) the expression of which is given in [8]. These effects will not be discussed further since they occur at frequencies outside the frequency range of interest here.

Once these two parameters computed, the dynamic mass density and compressibility of porous composite are computed using the following standard relationships:

\[ \tilde{\rho}_{pc} = \frac{\eta_0}{j \Pi_{pc}} \]
\[ \tilde{K}_{pc} = \frac{\gamma P_0/\phi_{pc}}{\gamma - 1/2 (\gamma - 1) \Pi_{pc}/\phi_{pc} \kappa} \]

\( \gamma \) is the ratio of the specific heats, \( \kappa \) is the thermal conductivity of the saturating air, and \( P_0 \) is the atmospheric pressure. Here \( \phi_{pc} \) is the total porosity of the porous composite, \( \phi_{pc} = \phi_p \phi_C + (1 - \phi_p) \phi_H \).

4. Modeling of the effective elastic properties

This mass density and bulk modulus corresponds to \( \tilde{\rho}_{mat} \) and \( \tilde{K}_{mat} \) and can be inserted into Equation 3 to Equation 8 to compute the dissipation inside the fluid phase of the porous composite. The next section presents the determination of the effective elastic properties in order to compute the structural dissipation inside the solid phase.

\[ E_L = E_C \phi_p + E_H (1 - \phi_p) + \ldots \]
\[ 4 (\mu_C - \mu_H) \phi_p (1 - \phi_p) \]
\[ \phi_p / K_H + (1 - \phi_p) / K_C + 1 / G_H \]
\[ 2 \]
\[ 1 / 2 K_L + 1 / 2 G_{TT} + 2 \nu_{TT} / E_L \]

where the subscripts \( H \) and \( C \) stands for the Host and the Client materials respectively. \( K_H \) and \( K_C \) are the lateral compression moduli and \( G_{TT} \) is shear modulus for the Host material. \( K_L \) is the lateral compression modulus of the composite and \( G_{TT} \) is a modified shear modulus introduced to simplify the expressions.

In addition, the mass density and the structural loss factor of the porous composite are given using a
mixing law as:

\[ \rho_{1, PC} = \phi_p \rho_{1,C} + (1 - \phi_p) \rho_{1,H} \]  
\[ \eta_{PC} = \phi_p \eta_C + (1 - \phi_p) \eta_H \]  

(17) \hspace{1cm} (18)

Knowing the elastic properties of the Host and of the Client, the structural effects can be introduced using the coupled poroelasticity equations as given in Equation 1 and Equation 2. In particular, the bulk modulus of the solid phase and Biot’s elastic coefficient \( \tilde{\rho} \) are then given by

\[ K_b = \frac{2}{3} G_{LT} \frac{1 + \nu_{LT}}{1 - 2\nu_{LT}} \]  
\[ \tilde{P} = (1 - \phi_{pc})^2 \tilde{K}_{pc} + K_b + \frac{4}{3} G_{LT} \]  

(19) \hspace{1cm} (20)

5. Experimental data

To provide with reliable data, a full characterisation of the acoustic and elastic material parameters has been realised. These procedures, presented in the following, have been applied only on the homogeneous porous materials, i.e. taken before arranging them as porous composites.

5.1. Characterisation of acoustic parameters

The characterisation of the acoustic parameters consists in the determination of the parameters of the JCAL model as described following the method described in [20, 21, 22]. This method relies on measured data of the dynamic mass density and of the dynamic bulk modulus of the material. These data are provided using a modified impedance tube using three microphone positions, two upstream and one directly placed at the rear side of the material as shown in Figure 3 [23].

The analytical determination of the parameters of the JCAL model further requires the prior knowledge of the static air flow resistivity \( \sigma \) and of the open porosity \( \phi \) of the material. These two quantities can be directly measured with a good accuracy using the corresponding ISO standard [24] for \( \sigma \) and using the method described by Beranek in [25] and further modified by Champoux et al. [26] for the open porosity \( \phi \).

In total, the procedure allows the determination of the following six parameters: the static air flow resistivity \( \sigma \) (in N.s.m\(^{-4} \), directly measured), the open porosity \( \phi \) (dimensionless, directly measured), the high frequency limit of the dynamic tortuosity \( \alpha_{\infty} \) (dimensionless, analytically determined), respectively the viscous and thermal characteristic lengths \( \Lambda \) and \( \Lambda' \) (both in m, analytically determined), and the static thermal permeability \( \Theta_0 \) (in m\(^2 \), sometimes also denoted \( k'_0 \), analytically determined).

It should be underlined that this method provides the independent expressions of the acoustic parameters, except for the expression of \( \Lambda \) which still depends on \( \alpha_{\infty} \). Another consequence is that the visco-inertial effects from one hand and the thermal effects from the other hand are characterised in a separate manner.

5.2. Characterisation of the elastic and damping parameters

The elastic and damping parameters have been estimated using a method inspired by the works of Langlois and co-workers in [27] which assumes an isotropic behaviour of the material. As for the acoustic parameters, this method has been applied on the non-perforated porous materials.

The basic idea of this method is to reproduce a mass/spring system where the mass, and thus the stress, is prescribed and the spring is represented by the porous material (see figure 4). Using charts of pre-computed results of the system response for different values of stiffness, the apparent Young’s modulus \( E_{\text{app}} \) of the spring, that is of the porous material, is identified. The loss factor \( \eta_p \) is estimated by a -n dB bandwidth method.

Moreover, \( E_{\text{app}} \) depends on the shape factor and on the Poisson’s ratio \( \nu \) of the tested sample. For brick-like samples, the shape factor is defined as half the radius to thickness ratio \( (R_{\text{tube}}/2d) \). Following [27], by testing several samples having different values of the shape factor, the “true” Young’s modulus and the Poisson’s ratio can be determined. In the present
study, brick-like samples of different thickness have been used.

6. Results and discussion

The model has been implemented using the parameters characterised and reported in Table I. Sound absorption coefficients and sound transmission loss data have been measured using the impedance tube shown in Figure 3. For the absorption, the measurement conditions correspond to those of the ISO 10534-2 \[28\].

Due to formatting restrictions, only one arrangement is presented here (other material associations will be presented during the conference). It is composed of a 29 mm diameter inclusion of Client material in a 46 mm diameter sample of Host material (see insert in the following figures). Results are shown for three different configurations and are compared to simulations or measurements when available obtained for the Host and for the Client material taken separately and considered as homogeneous.

In Figure 5, a satisfying correspondence is observed between simulation and measurement for the porous composite material. The damping predicted by the model seems to over-estimates that measured. This may be due to additional damping due to the actual mounting conditions of the sample inside the tube and at the interface between the inclusion and the Host material. One could note that the performances for this latter material compare favourably to those obtained for the homogeneous materials.

Figure 6 shows the performance of the same porous composite measured in transmission. A fair correspondence is observed on these results. These performances lie in the interval of the performances of Host and of the Client considered separately.

To prove the efficiency of the approach for multi-layer configurations, the porous composite has been covered with an impervious screen (see Table I). For this system, measurement and simulation compare well, expect in the frequency range where the quarter wavelength resonance occurs.

7. CONCLUSIONS

Based on Biot’s principle, an analytical approach has been proposed to model the vibro-acoustic response of the porous composite materials. The model has been implemented in TMM algorithm to simulate multi-layer configurations. Measurements and simulations presented above proved that these materials could represent an in-
teresting compromise between both sound absorption and sound transmission performances. With this incrementation, the proposed model broadens the range of applications of porous composites to multi-physics problems: acoustic and thermal performances, acoustic and mechanical properties, ... 

Further developments are in progress to fully account for the transverse isotropic behaviour of the composite materials studied.

References


